$$\lim_{n=\infty} (1+x)^{y_n}=(1+x)^y.$$

Hence we have $F(y)=(1+x)^y$; that is,

$$(1+x)^y=1+\frac{y}{1\,!}x+\frac{y(y-1)}{2!}x^2+\dots \quad |x|<1.$$

Thus the binomial theorem is proved for all finite values of y.

MOMENT OF INERTIA OF A RING CALCULATED BY AN ELEMENTARY METHOD.

By B. H. BROWN, Whitman College, Walla Walla, Washington.

The problem of finding the moment of inertia of a homogeneous circular ring, of unit density and of radius R with cross-section πr^2, when referred to a diameter, appears to be quite generally avoided by our writers on Elementary Mechanics. The following solution may prove of interest.

Suppose the ring to be edgewise to the plane of the paper and bisected by the plane of the paper giving the two circles of the figure as cross-sections of the ring. Plane PQ perpendicular to the paper bisects the ring and contains the axis of rotation, PQ passing through O, the center of the ring. The moment of inertia of the central section containing the plane PQ and of thickness dy is given by

$$I_c=\frac{\pi}{4}(A^4-a^4)dy. \quad \text{But } A=R+(r^2-y^2)^{\frac{1}{2}} \text{ and } a=R-(r^2-y^2)^{\frac{1}{2}}.$$

$$\therefore I_c=\frac{\pi}{4}\{[R+(r^2-y^2)^{\frac{1}{2}}]^4-[R-(r^2-y^2)^{\frac{1}{2}}]^4\}dy$$

$$=2\pi R^3(r^2-y^2)^{\frac{1}{2}}\,dy+2\pi R(r^2-y^2)^{\frac{3}{2}}\,dy.$$

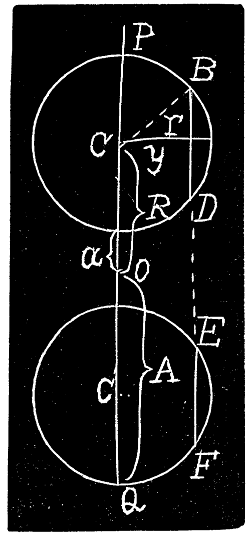
The moment of inertia for any other section as BF parallel to PQ at a distance y from the axis is of course given by $I_y=I_c+my^2$.

The area $BDEF$ cut from the ring by plane BF is equal to

$$\pi\,[R+(r^2-y^2)^{\frac{1}{2}}]^2-\pi\,[R-(r^2-y^2)^{\frac{1}{2}}]^2 \text{ or } 4\pi R(r^2-y^2)^{\frac{1}{2}},$$

and m equals this quantity multiplied by dy. Then

$$I_y = I_c + my^2 = 2\pi R^3 (r^2 - y^2)^{\frac{1}{2}}\, dy + 2\pi R (r^2 - y^2)^{\frac{3}{2}}\, dy + 4\pi R y^2 (r^2 - y^2)^{\frac{1}{2}}\, dy.$$



For the whole ring the formula becomes

$$I_w = \int_{-r}^{+r} [2\pi R^3 (r^2 - y^2)^{\frac{1}{2}} + 2\pi R (r^2 - y^2)^{\frac{3}{2}} + 4\pi R y^2 (r^2 - y^2)^{\frac{1}{2}}]\, dy = \pi^2 r^2 R [R^2 + \tfrac{5}{4} r^2] \ldots (1).$$

Since $2\pi^2 r^2 R$ = mass of ring, M, $I_w = \frac{M}{2}[R^2 + \frac{5}{4}r^2]$.

From (1), when $R=0$, $I_w=0$, the reversed moment of inertia of the inner portion of the ring exactly balancing that of the outer portion as the inner part backs over the center and becomes equal in mass to the outer part. This interesting hypothetical process by which the ring converts itself into two coinciding spheres with annulment of moment of inertia may perhaps be better appreciated by finding the moments of inertia of the outer and inner parts separately.

Take as the outer portion of the ring the part outside a cylindrical shell of radius R, with axis at O and perpendicular to the plane PQ. The inner part referred to below will be within this cylinder.

As before, $I_0 = \frac{\pi}{4}(A_1^4 - a_1^4)\, dy$, only in this case $a = R$.

$$\therefore I_0 = \frac{\pi}{4}\{[R + (r^2 - y^2)^{\frac{1}{2}}]^4 - R^4\}\, dy.$$

As before, after making obvious changes,

$$my^2 = \pi y^2 [2R (r^2 - y^2)^{\frac{1}{2}} + r^2 - y^2]\, dy.$$

Then

$$I_{O_1} = I_0 + my^2 = \frac{\pi}{4}\int_{-r}^{+r} [4R^3 (r^2 - y^2)^{\frac{1}{2}} + 6R^2 (r^2 - y^2) + 4R^2 (r^2 - y^2)^{\frac{3}{2}}$$

$$+ r^4 - 2r^2 y^2 + y^4]\, dy + \pi \int_{-r}^{+r} [2R (r^2 - y^2)^{\frac{1}{2}} + r^2 - y^2]\, y^2\, dy.$$

$$\therefore I_{O_1} = \frac{\pi}{2}[\pi r^2R^3 + \tfrac{5}{4}\pi r^4R + 4r^3R^2 + \tfrac{16}{15}r^5]$$

$$= \frac{\pi^2 r^2 R}{2}[R^2 + \tfrac{5}{4}r^2] + [2\pi r^3R^2 + \tfrac{8}{15}\pi r^5] \ldots (2).$$

It may be noted that (2) is equal to one-half the moment of inertia of the entire ring, plus the quantity $2\pi r^3R^2 + \frac{8}{15}\pi r^5$. When, in this case, $R=0$, the outer portion evidently becomes a sphere of radius r, and I_{O_1} becomes $\frac{8}{15}\pi r^5$, or $\frac{2}{5}Mr^2$, where M is the mass of the resulting sphere. This, of course, is as it should be.

Similar computations for the inner portion of the ring give:

$$I_i = \frac{\pi^2 r^2 R}{2}[R^2 + \tfrac{5}{4}r^2] - [2\pi r^3R^2 + \tfrac{8}{15}\pi r^5] \ldots (3).$$

When $R=0$, the inner portion of the ring backs across the center, "turning wrong side out" in becoming a sphere, and its moment of inertia is $-\frac{8}{15}\pi r^5$, or $-\frac{2}{5}Mr^2$. It may be noticed that, when $R=0$, the value of I for the inner portion of the ring is the same as that for the outer portion, but with its sign changed. On comparing (2) and (3), it is seen that their sum is equal to (1) for $R>r$ and $R=r$, and that their sum is equal to zero when $R=0$. Between $R=r$ and $R=0$, equation (1) passes through a series of complications which are interesting, but perhaps more so to the mathematician than to the experimental physicist.

DEPARTMENTS.

SOLUTIONS OF PROBLEMS.

ALGEBRA.

365. Proposed by C. N. SCHMALL, New York City.

In still water, a steam tug goes 6 miles an hour less when towing a barge than when alone. Having drawn the barge 30 miles up a stream, whose current runs 1 mile an hour, it returns alone and completes the journey in 12 8/11 hours. Find the rate of the tug in still water.

Solution by MARY INGRAM, Lucus (Kansas) High School.

Let x=rate of the tug in still water; $x-7$=rate up stream with barge; $x+1$=rate down stream.